1991 AB

1a. $f' = \frac{24x^2}{2} - 18x + c$

$f'(1) = -6$

$-6 = 12(1)^2 - 18(1) + c$

$c = 0$

$f' = 12x^2 - 18x$

since horizontal tangent

$m = 0$

$0 = 12x^2 - 18x$

$0 = 6x(2x-3)$

$x = 0 \quad x = \frac{3}{2}$

b. $f' = 12x^2 - 18x$

$f = \int f' = \frac{12x^3}{3} - \frac{18x^2}{2} + c$

$= 4x^3 - 9x^2 + c$

$f(2) = 0$

$0 = 4(2)^3 - 9(2)^2 + c$

$c = 4$

$f(x) = 4x^3 - 9x^2 + 4$

c. $\dfrac{\int_1^3 4x^3 - 9x^2 + 4}{3-1} = 5$

2a. $\int_0^1 \left[((1 + \sin \pi x) - x^2 \right] dx = 1.3$

b. $\int_0^1 \pi \left((1 + \sin \pi x)^2 - (x^2)^2 \right) dx$

c. $2\pi \int_0^1 x((1 + \sin \pi x) - x^2)\, dx$

3. $f' = \frac{3}{2}(1 + \tan x)^{\frac{1}{2}} \sec^2 x$

$f'(0) = \frac{3}{2}(1 + \tan 0)^{\frac{1}{2}} \sec^2 0$

$= \frac{3}{2}$

To write eqn. $x=0$ we must find y

$y = f(0) = (1 + \tan 0)^{\frac{3}{2}} = 1$

Eqn $\frac{3}{2} = \frac{y-1}{x-0}$

$\frac{3}{2}x = y - 1$

b. $\frac{dy}{dx} = f'(x)$

$f(0.02) = f(0) + dy$

$= y + dy$

$dy = f'(x)dx$

$= f'(0)(0.02)$

$= \frac{3}{2}(.02)$

$= .03$

$f(0.02) = f(x) + dy$

$= y + dy$

$= 1 + .03 = 1.03$

To check

$f(0.02) = (1 + \tan(0.02))^{\frac{3}{2}} = 1.03$

c. $x = (1 + \tan y)^{\frac{3}{2}}$

$x^{\frac{3}{2}} = 1 + \tan y$

$x^{\frac{3}{2}} - 1 = \tan y$

$\tan^{-1}(x^{\frac{2}{3}} - 1) = y$

$f^{-1}(x) = \tan^{-1}(x^{\frac{2}{3}} - 1)$

1991 #4

a. $|x|-2=0$

$|x|=2$

$\cancel{x=2}$ $\quad x=-2$

$\uparrow$

denom$=0$

b & c

$$f(x)=\frac{|x|-2}{x-2}$$

$$f'=\frac{(x-2)\frac{x}{|x|}(1)-(|x|-2)(1)}{(x-2)^2}$$

$$f'(1)=\frac{(1-2)\frac{1}{|1|}-(|1|-2)}{(1-2)^2}=0$$

$$f'(-1)=\frac{(-1-2)\frac{(-1)}{|-1|}(1)-(|-1|-2)(1)}{(-1-2)^2}$$

$$=\frac{4}{9}$$

d. $$f(x)=\frac{|x|-2}{x-2}$$

substitute values

$$\frac{|.1|-2}{.1-2}=\frac{-1.9}{-1.9}=1 \qquad \frac{|1000|-2}{1000-2}=+1$$

$-1 \leq Y \leq 1$

#5

f'

0		1		2		3
x	−	0	−	x	+	

f''

0		1		2		3
x	+	0	−	x	−	
	up		down		down	

a. max $x=0$

min $x=-2$, $x=2$

b. Pts. of infl.

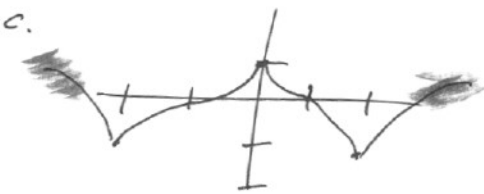
$x=1$

$x=-1$

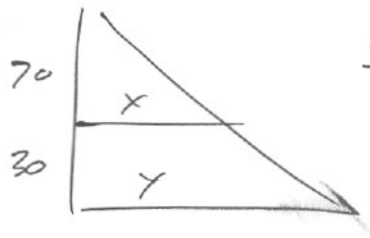
f'' changes from + to − at $x=1$

since symmetric also at $x=-1$

c.



1991 #6



$\frac{dx}{dt} = 2$

$\frac{dy}{dt} = ?$

a. Triangles are similar

$$\frac{70}{x} = \frac{70+30}{Y}$$

$$70y = 100x$$

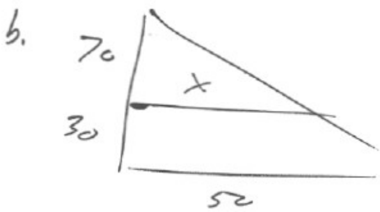
$$7y = 10x$$

$$y = \frac{10}{7}x$$

$$\frac{dy}{dt} = \frac{10}{7}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{10}{7}(2) = \frac{20}{7} = 2.85 \text{ ft/sec}$$

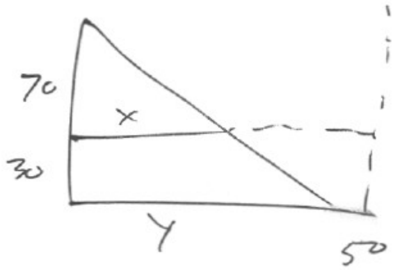
b.

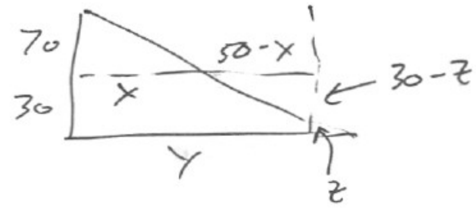


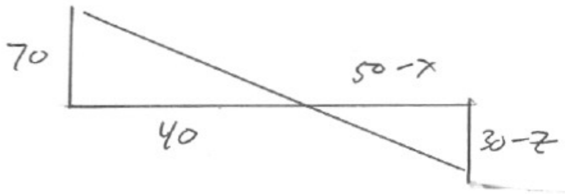
$$\frac{70}{x} = \frac{100}{50}$$

$$x = 35$$

c.







$$\frac{70}{x} = \frac{30-z}{50-x}$$

$$\frac{50-x}{x} = \frac{30-z}{70}$$

$$3500 - 70x = 30x - zx$$

$$3500 - 100x + zx = 0$$

$$3500 - 100x = -zx$$

$$-\frac{3500}{x} + 100 = z$$

$$-3500x^{-1} + 100 = z$$

$$3500x^{-2}\frac{dx}{dt} = \frac{dz}{dt}$$

$$\frac{3500}{x^2}\frac{dx}{dt} = \frac{dz}{dt}$$

$$\frac{3500}{(40)^2}(2) = \frac{dz}{dt}$$

$$\frac{35}{8} = \text{ft/se}$$

1991 BC 1

1a. $x(t) = \int v(t)$

$= \int 12t^2 - 36t + 15$

$= \frac{12t^3}{3} - \frac{36t^2}{2} + 15t + c$

$= 4t^3 - 18t^2 + 15t + c$

$x(1) = 0$

$0 = 4(1)^3 - 18(1)^2 + 15(1) + c$

$c = -1$

$\therefore x(t) = 4t^3 - 18t^2 + 15t$

b. At rest when $v = 0$

$12t^2 - 36t + 15 = 0$

$4t^2 - 12t + 5 = 0$

$(2t-5)(2t-1) = 0$

$t = \frac{5}{2} \quad t = \frac{1}{2}$

C. TO FIND MAX CHECK DERIV FOR CRITICAL PTS

$v' = 24t - 36$

$0 = 24t - 36$

$36 = 24t$

$\frac{3}{2} = t$

	$12t^2 - 36t + 15$
$\frac{3}{2}$	$= -12$
End pts → 0	$= 15$
→ 2	$= -81$

MAX VELOCITY 15

d. TOTAL DISTANCE

$v = 0$ AT $\cancel{t = \frac{5}{2}}$ $t = \frac{1}{2}$

OUTSIDE INTERVAL

	$x(t) = 4t^3 - 18t^2 + 15t - 1$
0	-1
$\frac{1}{2}$	2.5
2	-11

0 TO $\frac{1}{2}$ $\quad 2.5 - 1 = 3.5$

$\frac{1}{2}$ TO 2 $\quad -11 - 2.5 = |-13.5|$ $\quad 13.5$

$\overline{17}$ $\quad (17)$

2.a. Use sign chart for f'

$f' = x(-e^{1-x}) + e^{1-x}$

$= (-x+1)e^{1-x}$

		1	
$-x+1$	+ +	0	− −
e^{1-x}	+ +	+	+ +
f'	+	0	−

Inc $(-\infty, 1)$

b. Max would occur at $x = 1$ (from sign chart)

$f(1) = 1e^{1-1} = 1$ MAX AT $Y = 1$

c. PTS OF INFLECTION f'' GRAPH

$f'' = (-x+1)(-e^{1-x}) + e^{1-x}(-1)$

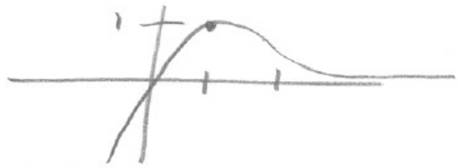
$= (x-1)(e^{1-x}) - e^{1-x}$

$= (x-1-1)(e^{1-x})$

$= (x-2)(e^{1-x})$

		2	
$x-2$	−	0	+
e^{1-x}	+ +	+	+ +
f''	−	0	+

PT OF INFLECTION AT $x = 2$

d. 

1991 BC 3

3\. a.

FIND INTERSECTION

$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$$

$$\sin x + \cos x \Big]_0^{\frac{\pi}{4}}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0+1)$$

$$= \frac{2\sqrt{2}}{2} - 1 = \sqrt{2} - 1 = .4142$$

b. $$\pi \int_0^{\frac{\pi}{4}} (\cos x)^2 - (\sin x)^2\,dx$$

$$= \pi \int_0^{\frac{\pi}{4}} (\cos x)^2 - (1 - \cos^2 x)\,dx$$

$$= \pi \int_0^{\frac{\pi}{4}} \cos^2 x - 1 + \cos^2 x = \frac{1}{2}\pi$$

$$= 1.5708$$

3c. Add together squares

one side $= \sin x - \cos x$

Area of sq. $= (\sin x - \cos x)^2$

$$\therefore \int_0^{\frac{\pi}{4}} (\sin x - \cos x)^2\,dx = .285$$

4a. $F'(x)$ is derivative of Integral

$$= \sqrt{(2x)^2 + (2x)}\,(2)$$

b. $(2x)^2 + 2x > 0$

$4x^2 + 2x > 0$

$2x(x+1) > 0$

	-1	0
$2x$	$-$	0
$x+1$	$-\ 0$	
6	$+\ 0$	$-\ 0\ +$

DOMAIN $x \le -1$ $x \ge 0$

c. $$\lim_{x \to \frac{1}{2}} \sqrt{4x^2 + 2x}\,(2)$$

$$\sqrt{4(\tfrac{1}{2})^2 + 2(\tfrac{1}{2})}\,(2) = \sqrt{1+1}\,(2) = 2\sqrt{2}$$

d.